Pearson Edexcel Level 3

GCE Mathematics

Advanced Level

Paper 1 or 2: Pure Mathematics

Practice Paper B	**Paper Reference(s)**
Time: 2 hours	**9MA0/01 or 9MA0/02**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 13 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1. Use proof by contradiction to prove the statement: 'The product of two odd numbers is odd.'

(5 marks)

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2. (a) Prove that the sum of the first n terms of an arithmetic series is $S = \frac{n}{2}\left(2a + (n-1)d\right)$.

(3 marks)

(b) Hence, or otherwise, find the sum of the first 200 odd numbers.

(2 marks)

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3. A curve has the equation $y = \ln 3x - \mathrm{e}^{-2x}$.

Show that the equation of the tangent at the point with an x-coordinate of 1 is

$$y = \left(\frac{\mathrm{e}^2 + 2}{\mathrm{e}^2}\right)x - \left(\frac{\mathrm{e}^2 + 3}{\mathrm{e}^2}\right) + \ln 3.$$

(6 marks)

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4. The curve C has parametric equations $x = 7\sin t - 4$, $y = 7\cos t + 3$, $-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{3}$.

(a) Show that the cartesian equation of C can be written as $(x+a)^2 + (y+b)^2 = c$, where a, b and c are integers which should be stated.

(3 marks)

(b) Sketch the curve C on the given domain, clearly stating the endpoints of the curve.

(3 marks)

(c) Find the length of C. Leave your answer in terms of π.

(2 marks)

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5. The coordinates of A and B are $(-1, 7, k)$ and $(4, 1, 10)$ respectively. Given that the distance from A to B is $5\sqrt{5}$ units,

(a) find the possible values of the constant k.

(3 marks)

(b) For the larger value of k, find the unit vector in the direction of $\overrightarrow{OA}$.

(3 marks)

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6. Given that $\int_{\ln 2}^{\ln b}\left(\frac{e^{2x}}{e^{2x}-1}\right)dx = \ln 4$, find the value of b showing each step in your working.

(8 marks)

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7. A sequence is given by $x_1 = 4$, $x_{n+1} = px_n - 9$ where p is an integer.

(a) Show that $x_3 = 4p^2 - 9p - 9$.

(2 marks)

Given that $x_3 = 46$,

(b) find the value of p.

(3 marks)

(c) Hence find the value of x_5.

(1 mark)

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8. Express $\frac{6}{4x^2 + 8x - 5} + \frac{3x+1}{2x-1}$ as a single fraction in its simplest form.

(4 marks)

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9. The diagram shows the graph of h(x).

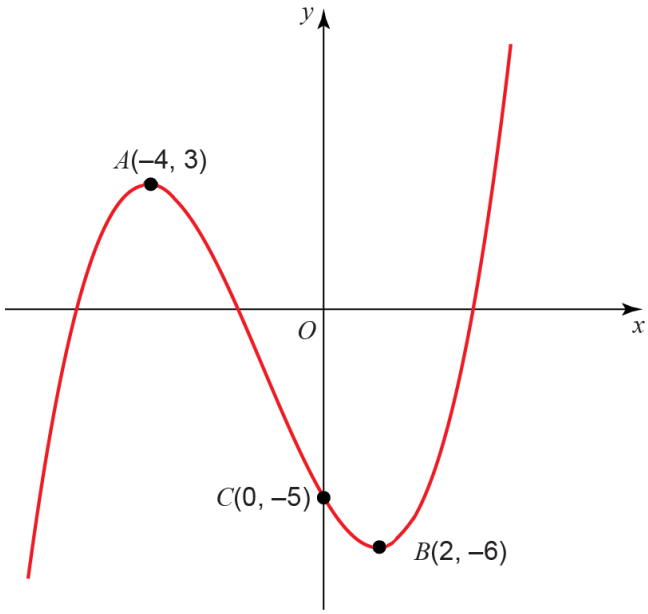

Figure 1

The points $A(-4, 3)$ and $B(2, -6)$ are turning points on the graph and $C(0, -5)$ is the y-intercept. Sketch on separate diagrams, the graphs of

(a) $y = |\mathrm{f}(x)|$

(3 marks)

(b) $y = \mathrm{f}(|x|)$

(3 marks)

(c) $y = 2\mathrm{f}(x + 3)$

(3 marks)

Where possible, label clearly the transformations of the points A, B and C on your new diagrams and give their coordinates.

10.

$$g(x) = \frac{2}{x - 1} - e^x$$

(a) By drawing an appropriate sketch, show that there is only one solution to the equation $g(x) = 0$.

(2 marks)

(b) Show that the equation $g(x) = 0$ may be written in the form $x = 2e^{-x} + 1$.

(2 marks)

Let $x_0 = 1.5$.

(c) Use the iterative formula $x_{n+1} = 2e^{-x_n} + 1$ to find to 4 decimal places the values of x_1, x_2, x_3 and x_4.

(2 marks)

(d) Using $x_0 = 1.5$ as a first approximation, apply the Newton–Raphson procedure once to $g(x)$ to find a second approximation to α, giving your answer to 4 decimal places.

(4 marks)

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11. (a) Find the binomial expansion of $\frac{1+x}{\sqrt{1 - 2x}}$ in ascending powers of x up to and including the x^2 term, simplifying each term.

(4 marks)

(b) State the set of values of x for which the expansion is valid.

(1 mark)

(c) Show that when $x = \frac{1}{100}$, the exact value of $\frac{1+x}{\sqrt{1 - 2x}}$ is $\frac{101\sqrt{2}}{140}$.

(2 marks)

(d) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (a) and hence obtain an approximation to $\sqrt{2}$. Give your answer to 5 decimal places.

(3 marks)

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12. The diagram shows the curve with equation $y = \frac{1}{2}x^3\sqrt{4-x^2}$

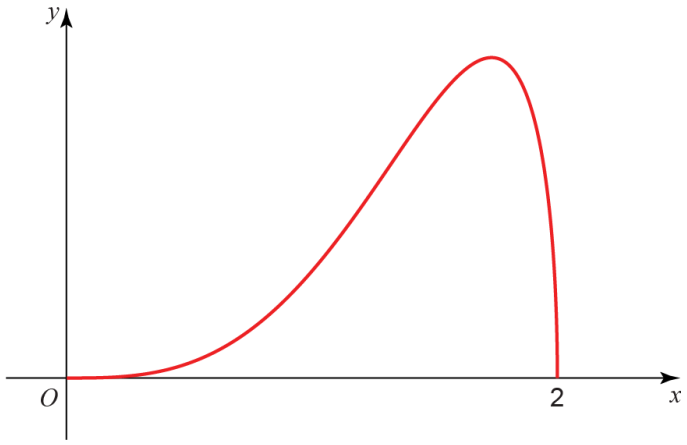

Figure 2

(a) Complete the table with the value of y corresponding to $x = 1.5$. Give your answer correct to 5 decimal places.

x	0	0.5	1	1.5	2
y	0	0.12103	0.86603		0

(1 mark)

Given that $I = \int_0^2 \left(\frac{1}{2}x^3\sqrt{4-x^2}\right)\mathrm{d}x$,

(b) use the trapezium rule with 4 equal width strips to find an approximate value of I, giving your answer to 4 significant figures.

(3 marks)

(c) By using an appropriate substitution, or otherwise, find the exact value of $\int_0^2 \left(\frac{1}{2}x^3\sqrt{4-x^2}\right)\mathrm{d}x$, leaving your answer as a rational number in its simplest form.

(6 marks)

(d) Suggest one way in which your estimate using a trapezium rule could be improved.

(1 mark)

13. (a) Express $5\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \pi$. Write R in surd form and give the value of α correct to 4 decimal places.

(4 marks)

The temperature of a kiln, T °C, used to make pottery can be modelled by the equation $T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right)$, $0 \leqslant x \leqslant 72$, where x is the time in hours since the pottery was placed in the kiln.

(b) Calculate the maximum value of T predicted by this model and the value of x, to 2 decimal places, when this maximum first occurs.

(4 marks)

(c) Calculate the times during the first 24 hours when the temperature is predicted, by this model, to be exactly 1097 °C.

(4 marks)

TOTAL FOR PAPER IS 100 MARKS

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